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Agricultural Irrigation Initiative: Precision Water Application Test

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Executive Summary

As water becomes increasingly scarce, agricultural management of this precious resource becomes ever more critical. Maximizing performance of agricultural land and minimizing the use of water and other resources, such as energy, requires optimal use of new irrigation technologies such as Variable Rate Irrigation (VRI).

Ensuring efficient application of water to a field during irrigation helps to minimize water use, thereby also minimizing the use of energy required to apply that water. As part of its Agricultural Irrigation Initiative, the Northwest Energy Efficiency Alliance (NEEA) commissioned Oregon State University (OSU) to conduct an evaluation of the precision of as-applied water from VRI systems installed on center pivots. The overarching study goal was to determine the capability of the VRI system to deliver an irrigation prescription matched to the needs of the field. OSU researchers carried out the evaluation by:

- Creating a map of prescribed water application that presented the VRI pivot with a range of challenges
- Performing an extensive "catch-can test" experiment, which collected the actual water applied under a pivot using the created water application prescription map
- Comparing the measured, as-applied water depths to the prescribed depths
- Analyzing the pivot response to determine general advice to growers when they create prescriptions
- Creating a metric that accurately reflects the pivot's performance, and a method to predict the as-applied water, given the prescription
- Making recommendations for setup and optimizing performance

The pivot performed well in the test case and followed the prescribed pattern of water application over the entire test area. The research team measured a statistically significant under-application of water of seven percent, which could be due to evaporative loss or wind redistribution. Discrepancies between the prescribed water application and the measured water application occurred near transitions of water depth within the prescription (as expected). Based on an extensive mathematical analysis of these "transitions," the researchers identified a minimum size of a management zone of twenty-three meters (seventy-five feet); in other words, any area within a VRI prescription smaller than twenty-three meters (seventy-five feet) along any edge will not be managed independently. The result informs the formation of VRI prescriptions and supports the utility of creating sprinkler "banks" that are sufficiently large. The research team also concluded that verification of equipment performance is essential for proper operation.

These findings indicate that prescriptions with small isolated areas (in other words, a pixelated look) will not be applied exactly as prescribed due to the sprinkler geometry and wind redistribution. A prescription in which all independent areas are greater than twenty-three meters (seventy-five feet) along each edge will yield a better translation from software-to-field. The "convolution equation" in this report facilitates a prediction of the actual water application given the known prescription.

1. Introduction

As water becomes increasingly scarce, agricultural management of this precious resource becomes ever more critical. Maximizing performance of agricultural land and minimizing the use of water and other resources, such as energy, requires optimal use of new irrigation technologies such as Variable Rate Irrigation (VRI). Ensuring efficient application of water to a field during irrigation helps to minimize water use, thereby also minimizing the use of energy required to apply that water.

The Northwest Energy Efficiency Alliance (NEEA) launched the Agricultural Irrigation Initiative in 2011 with the goal of reducing agricultural irrigation energy use by twenty percent by 2020. NEEA is an alliance funded by more than 140 utilities and energy efficiency organizations in Idaho, Oregon, Montana, and Washington working to accelerate the innovation and adoption of energy-efficient products, services, and practices in the Northwest. As part of this Initiative, NEEA commissioned Oregon State University (OSU) to conduct an evaluation of the precision of as-applied water from VRI systems installed on center pivots. This report summarizes the findings of that evaluation.

VRI is a type of site-specific water resource management accomplished by dividing a field into discrete management zones that allow independent management of the water and fertilizer demands. Growers achieve differing irrigation levels within each management zone through an actuated ON/OFF duty cycle of the sprinkler nozzles with the water application depth specified as a percentage of the full-depth irrigation (determined by flow rate and speed of pivot travel).

This report is one in a series of twelve reports addressing specific areas of NEEA's Agricultural Irrigation Initiative. All twelve reports are available at http://neea.org/reports.

Given the industry-specific or scientific natures of some terms used in this report, please refer to the AgGateway AgGlossary (http://agglossary.org/wiki/index.php/main_page) for definitions.

1.1. Background

Several researchers[1] have tested the impacts of the "duty cycle" approach in a series of systematic precision water application tests ("catch-can" tests). Several studies have investigated the fidelity between the prescribed water depth and the depth actually applied by a VRI system. Perry et al. (2003) performed a catch-can test along a single radial line and found a detectable under-application of water, while the pattern of applied water followed the prescribed pattern in a qualitative manner. King et al. (2009) performed a catch-can test along a single radial line and tested both uniform and variable applications. They found a correlation coefficient of 0.9 between prescribed and measured quantities of both water and a surrogate fertilizer.

[1] Perry et al. (2003), Perry et al. (2004), King et al. (2005), and Dukes and Perry (2006)

O'Shaughnessy et al. (2011) performed a catch-can test along radial transects and along arcs of constant radius. They found that the coefficient of uniformity within management zones was acceptable (up to 0.88) for zones when application rates were 50% or greater. They also observed areas of overshoot, where the observed application depth was greater than expected after a prescribed step change in application depth. Finally, a series of catch-can tests were performed as a validation of an automated VRI approach, in which the prescribed water depth is adjusted in real time with data inputs (King et al. 1999; Kim and Evans 2009a; Kim et al. 2009b). These studies found correlations between the prescribed and measured water applications ranging from 0.96 to 0.98.

1.2. Objectives

The researchers identified several objectives for quantifying VRI performance and efficiency in this study:

- First, the study design and irrigation prescription attempted to identify the smallest achievable size of management zones.
- Second, the study design maximized the number of transitions of different magnitudes (between different irrigation depths) to address the effect of application depth on performance.
- Finally, researchers created a performance coefficient that expands the concept of the uniformity coefficient to the VRI pivot.

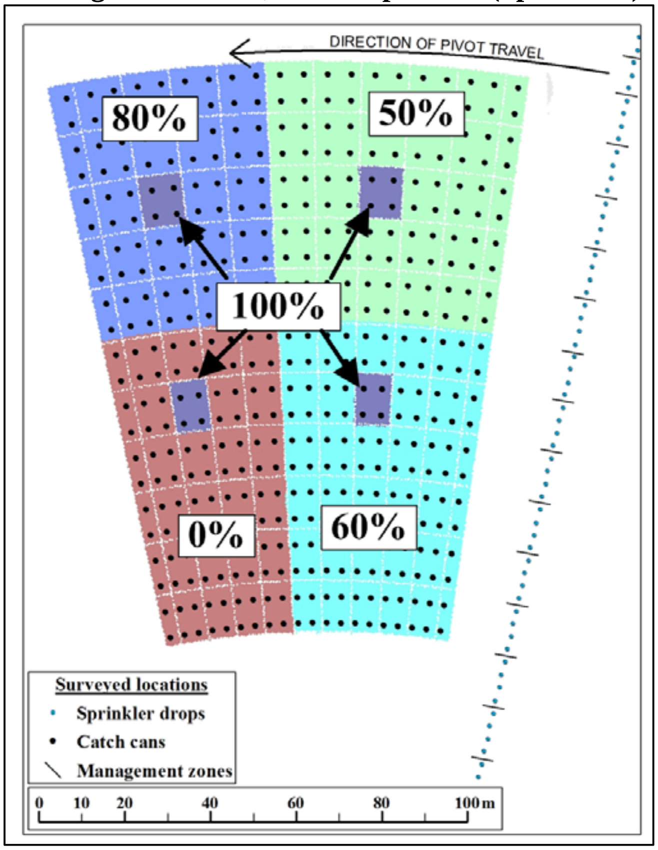
The overarching study goal was to determine the capability of the VRI system to deliver an irrigation prescription matched to the needs of the field.

2. Methodology

2.1. Test Description

Researchers performed the test using a nine-span Valley 8000 Series Pivot with a design flow rate of 3,539 liters per minute (LPM) (935 gallons per minute (GPM)).[2] In the fall of 2012, researchers retrofitted the pivot with a Valley Variable Rate Irrigation package with thirty management zones.[3] This VRI system uses solenoid valves to actuate the flow for groups of sprinklers. The number of nozzles per management zone varied from seven at the center to three at the pivot end. The researchers chose as the total number of management zones the maximum number of zones possible with this VRI system. The radial direction had management zones 12 meters (39.4 feet) in side length. Along concentric arcs, management zones ranged in arc length from 6.7 meters (22.0 feet) (at the innermost part of the study area) to 11.5 meters (37.7 feet) (at the outermost extent of the study area), as illustrated in Figure 1.

Figure 1. Surveyed Locations of Catch Cans, Pivot Management Zones, and Drop Tubes (Sprinklers)



Note: Location markers overlay the prescription, with percentage of full application.

[2] Liters per minute (LPM) is also referenced as "l min^{-1} "

[3] A management zone is defined as a single group of sprinklers actuated simultaneously.

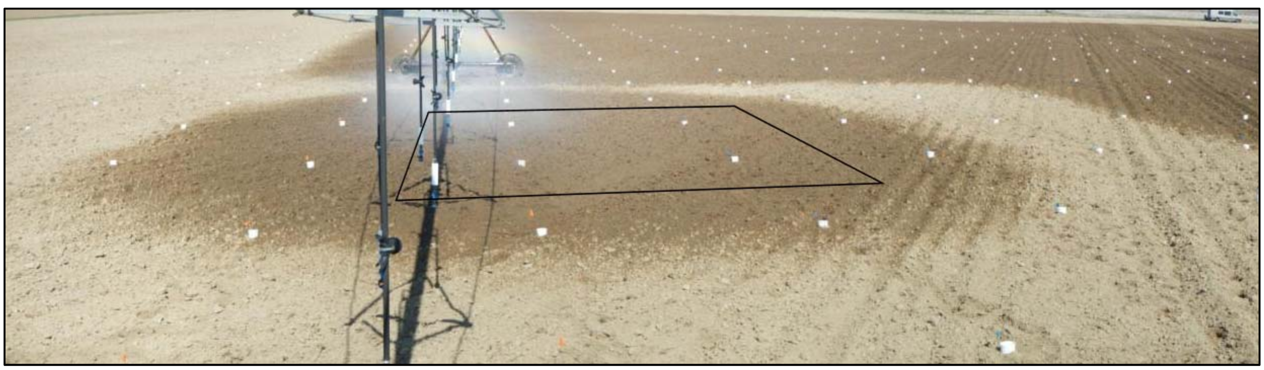
The manufacturer selected the grouping of nozzles within management zones based on its internal engineering analysis. Researchers equipped the pivot with Nelson R3000 sprinklers (brown plate) with nominal flow of 29.9 to 44.7 LPM (7.9 to 11.8 GPM) under the tested area. They attached the sprinklers to weighted drop tubes at 1.8 meters (5.9 feet). At this mounting height, the manufacturer reports a sprinkler throw of approximately 18.9 meters (62.0 feet). Each sprinkler had a Nelson Uni-Flo pressure regulator that limited the nozzle pressure to 103.4 kilopascals (15.0 psi).

2.2. Field Test Conditions

The researchers conducted this study in an agricultural field in Benton County, Washington. The field was plowed, disked, and harrowed prior to the experiment, leaving a smooth surface free of vegetation. Elevations within the test area ranged between 224.1 and 226.8 meters (735.2 and 744.1 feet) above sea level. Weather conditions were generally cold and humid with wind speeds ranging between 0.0 and 4.5 meters per second (0.0 and 10.1 miles per hour) (mean 1.7 meters per second (3.8 miles per hour)), temperatures ranging between 2^{o} C and 13^{o} C (36 ^{o} F and 55 ^{o} F), and relative humidity ranging from thirty to seventy percent.

Using the configuration shown in Figure 1, researchers placed 440 containers (2.1 liters (0.6 gallons) polypropylene, 152 mm (6 inches) tall and 152 mm (6 inches) in diameter) under the fifth, sixth, and seventh spans of the pivot to obtain consistent spacing in the radial direction. The management zones each had exactly three nozzles in the area in which the containers were located. The researchers spaced the containers 6.1 meters (20.0 feet) apart in the radial direction and one degree apart in the angular direction, with the entire array covering approximately one hectare of the field. A survey-grade differential GPS located the containers within 100 mm (3.9 inches) of their nominal positions. Figure 2 shows a photo of a section of the array.

Figure 2. Photo of a Segment of the Catch-Can Array



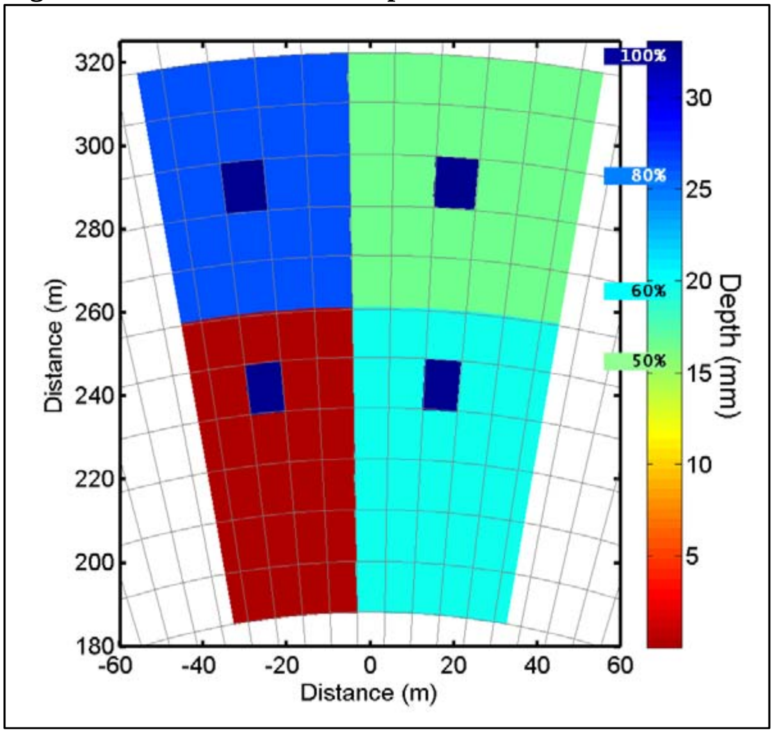
Notes: The area within the black square is set to full irrigation depth (100%) while the area immediately surrounding this box is prescribed to receive no water (0%). Radial lines of catch cans (white) proceed from foreground to background. Arcs of catch cans are oriented across the field of view.

2.3. Test Performance Details

The researchers performed two tests using identical prescriptions for irrigation depth, the map of which is shown in Figure 3. During both tests, they ran the pivot at ten percent of maximum speed with an expected nominal application depth of 33 mm (1.3 inches) (corresponding to 100%). Between the first and second test, they returned the pivot to its original position so that it traveled in the same counterclockwise direction (from right to left in the figures) for both tests. They positioned the pivot for both tests so that when movement began, the pivot was located more than two sprinkler throw radii away from the first catch can. The researchers ran the pivot until the observed sprinkler throw had traveled beyond the last line of catch cans. The researchers determined application depths by weighing each catch can.[4]

Prior to the experiment, researchers numbered each catch can and recorded the tare weight. Prior to each trial, they cleaned the catch cans of debris and insects, placed them in vertical orientation, and staked them in place. Researchers lidded each catch can once they observed it was no longer receiving water. After the experiment, they calculated the applied depth using measured mass, density of water, and diameter of the container's opening.

Figure 3. Prescribed Water Depths Given in cm



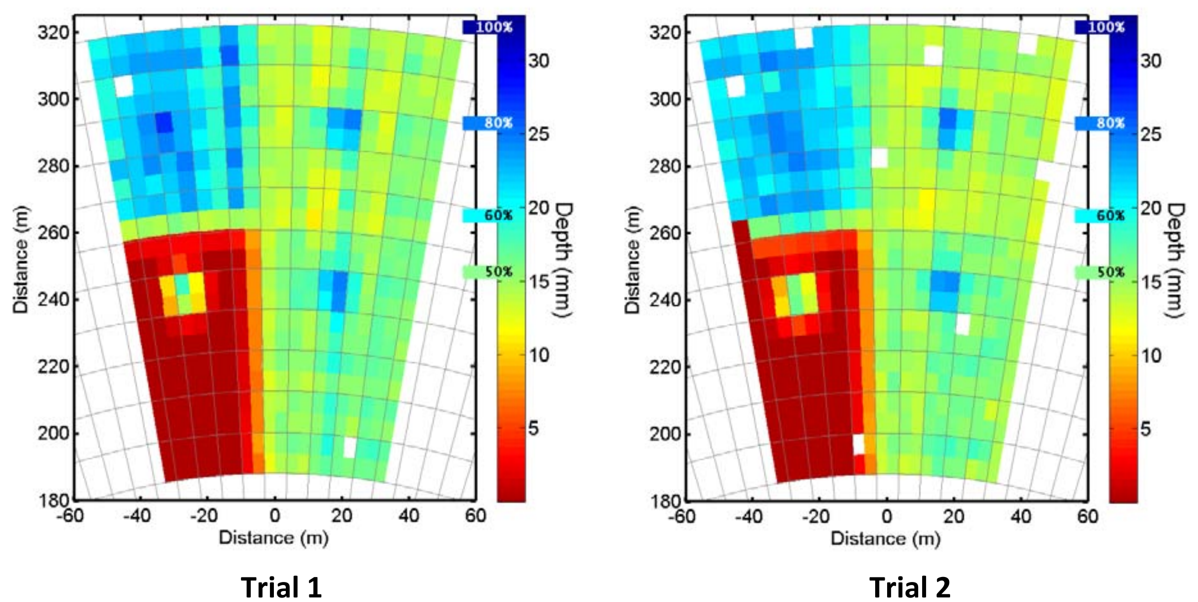
Notes: The prescription is divided into four quadrants: deep red 0% (dry), green 50% (16.5 mm/0.6 inches), turquoise 60% (19.8 mm/0.8 inches), and medium blue 80% (26.4 mm/1.0 inches). Within each quadrant an isolated management zone is set to full (100% - dark blue) irrigation, or 33 mm/1.3 inches.

[4] Using an Ohaus CL Series scale with 1 g resolution

3. Findings, Measurements, and Analysis

Figure 4 illustrates the measured application depths from Trials 1 and 2. Data points without color indicate missing data. Given the large number of catch cans, missing points and outliers accounted for less than one percent of the data in Trial 1, and two percent in Trial 2. The general pattern of applied water matches the pattern of the prescription in Figure 3 above.

Figure 4. Water Depths as Measured by the Catch Can Array for Trials 1 and 2

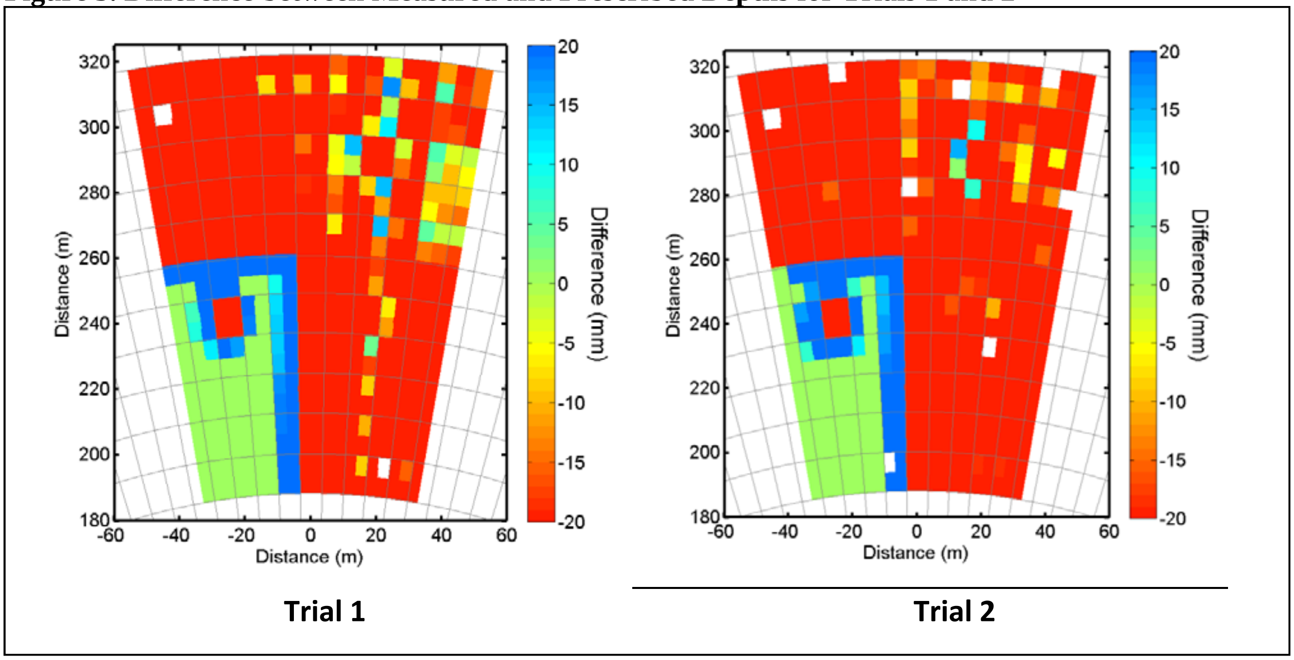


Overall, the measured water depth is less than the prescribed water depth in most areas. In particular, the prescribed 100% management zone within the area of 0% application is substantially under-applied. The applied water is approximately seven percent less than the prescribed amount when averaged over the entire array (statistically significant $p<0.05$ using a 2 tailed *t*-test). Perry et al. (2003) also found similar under-application for VRI systems.

The researchers performed additional statistical analysis on the data to verify that VRI delivered statistically different water depths across the experimental area as expected. They checked all permutations comparing regions of different depths (100%, 80%, 60%, 50% and 0%) and confirmed that all measured zones had statistically different depths at the $p<0.05$ level. Overall, the pivot was capable of providing statistically different water depths at ten percent intervals in application granularity.

Figure 5 presents the differences between the measured and prescribed irrigation over the area shown in the earlier image. The figure clearly shows that the areas of the field with the greatest difference between the measured and expected depths occur near the transitions from one prescribed depth to another. Further, the total magnitude of the disparity appears to be a function of total step change in prescribed water depth. This is apparent at the boundary between the 60% and 0% regions, and at the boundary between the 100% and 0% regions. Relative to the prescribed water depth, the former transition represents the greatest over-application of water, while the latter represents the largest under-application.

Figure 5. Difference between Measured and Prescribed Depths for Trials 1 and 2



Notes: Color scale is in mm. Negative values indicate under-application; positive values indicate over-application.

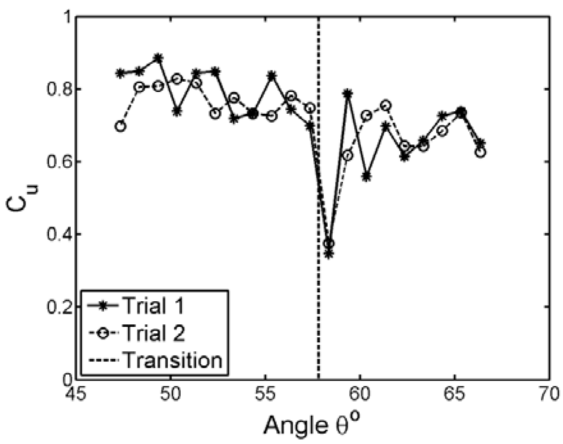
To describe the performance of the pivot in more quantitative terms, a new "performance coefficient" is derived following the approach of Heermann and Hein (1968), who developed the coefficient of uniformity for center pivot irrigation systems. By following their approach, and replacing the measured average depth with the prescribed water as the comparator, the following new coefficient can be defined:

$$C_p = 1 - \frac{\sum_i |r_i D_i - r_i P_i|}{\sum_i r_i D_i}, \tag{1}$$

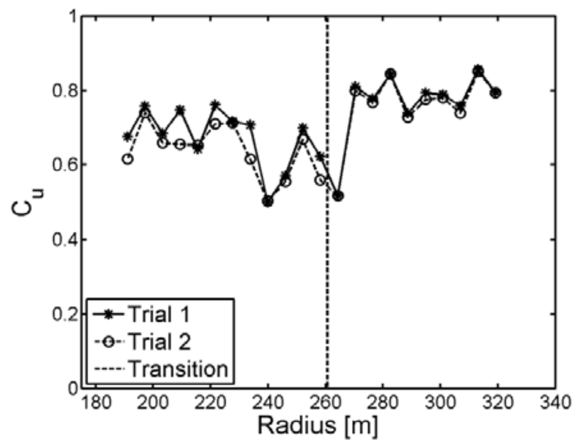
where r_i is the distance along the boom associated with each catch can, P_i is the prescribed water depth at each catch can location, and D_i is the measured water depth at each catch can location.

This formula assumes that the catch cans are equally spaced in the radial direction. Figure 6 presents plots of the performance coefficient as a function of radius and pivot angle and illustrates that the locations at significant transitions in prescribed water depths have the lowest performance values. This is expected, as no application system could perfectly reproduce the sharp transitions as drawn in the prescription due to the sprinkler patterns and the random nature of atmospheric redistribution.

Figure 6. Performance Coefficient as a Function of Pivot Angle and Radius



Function of Pivot Angle



Function of Radius

Notes: These coefficients show a strong degradation in performance near the points of transition (denoted by the vertical dashed line) between differently-prescribed water levels.

The research team performed a detailed mathematical analysis of the transition zones to determine the “smoothing” inherent in the translation from the idealized prescription and the as-applied water. They found the characteristic length scale of this smoothing process to be 4.15 m (13.6 feet) (see Appendix A for details). Once researchers know the degree of smoothing, they can predict the best possible rendition of the prescription by the pivot by convolving the prescription with a Gaussian whose standard deviation is 4.15 m (13.6 feet).

Figure 7 illustrates the result of this convolution process; this is the best achievable implementation of the original prescription (in Figure 3). Comparing this result with the raw data presented in Figure 4 shows that the locations of highest disagreement with the initial prescription now show much better agreement. The enhanced agreement between the best achievable implementation of the prescription and the measured water depths can be quantified with the performance coefficient in the following way,

$$C_p = 1 - \frac{\sum_i \left| r_i D_i - r_i P_{BEST,i} \right|}{\sum_i r_i D_i}, \qquad (2)$$

where $P_{BEST,i}$ is now the best achievable implementation of the prescription, as seen in Figure 7.

Figure 7. Smoothed Prescription

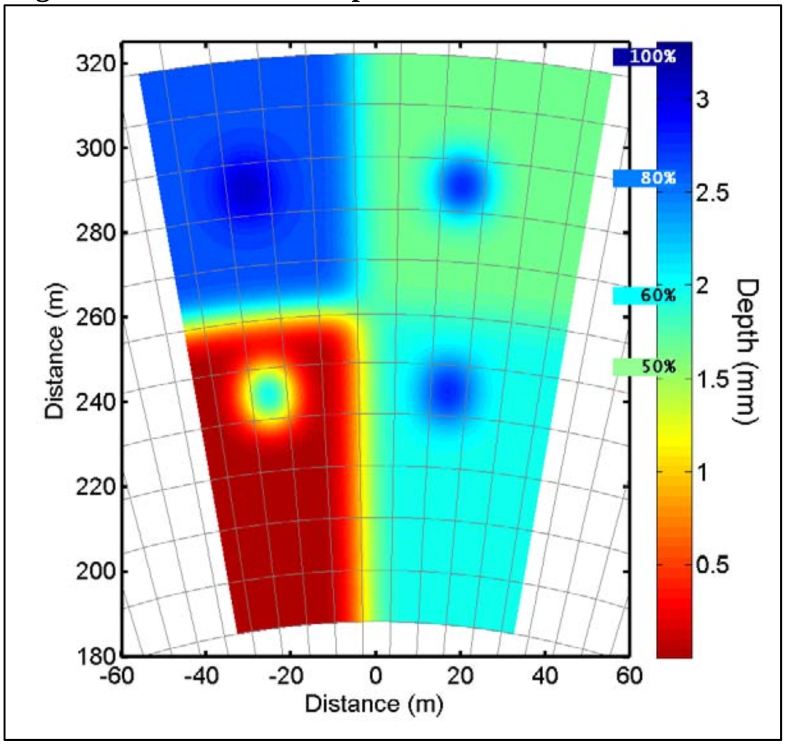
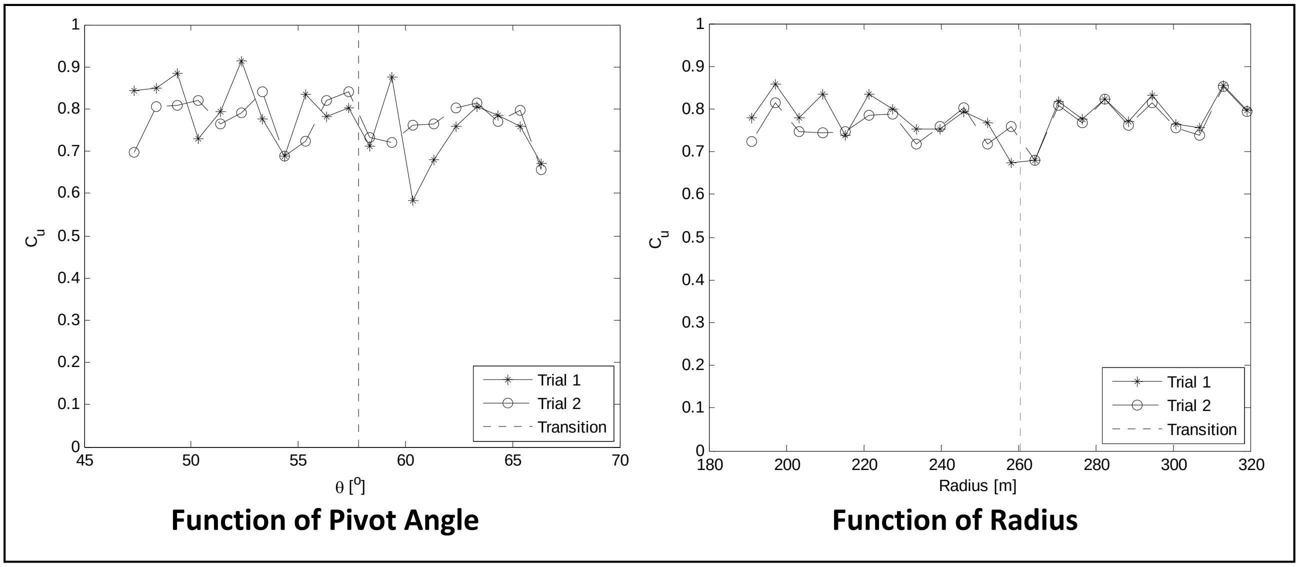


Figure 8 presents plots of the performance coefficient, calculated according to Equation 2. The low performance near transitions has disappeared, and all areas show a higher level of recorded performance.

Figure 8. Performance Coefficient as a Function of Pivot Angle and Radius



Notes: Calculated using the smoothed prescription shown in Figure 7. Transitions, identified above as the vertical dashed lines, are no longer identified as places of poor performance.

Returning to Figure 7, the original 100% application island (from Figure 3) in the lower left shows that this 100% application is no longer achievable given the smoothing that occurs during water application. This process results in a best achievable average application depth far less than that in the initial prescription. This best achievable water depth is in agreement with the measured water depth, as a comparison to Figure 4 indicates. The local under-application of water in this zone occurs because the 100% management zone is not independent of its neighboring management zones; in other words, a management zone of this size is too small to operate independently of its neighbors.

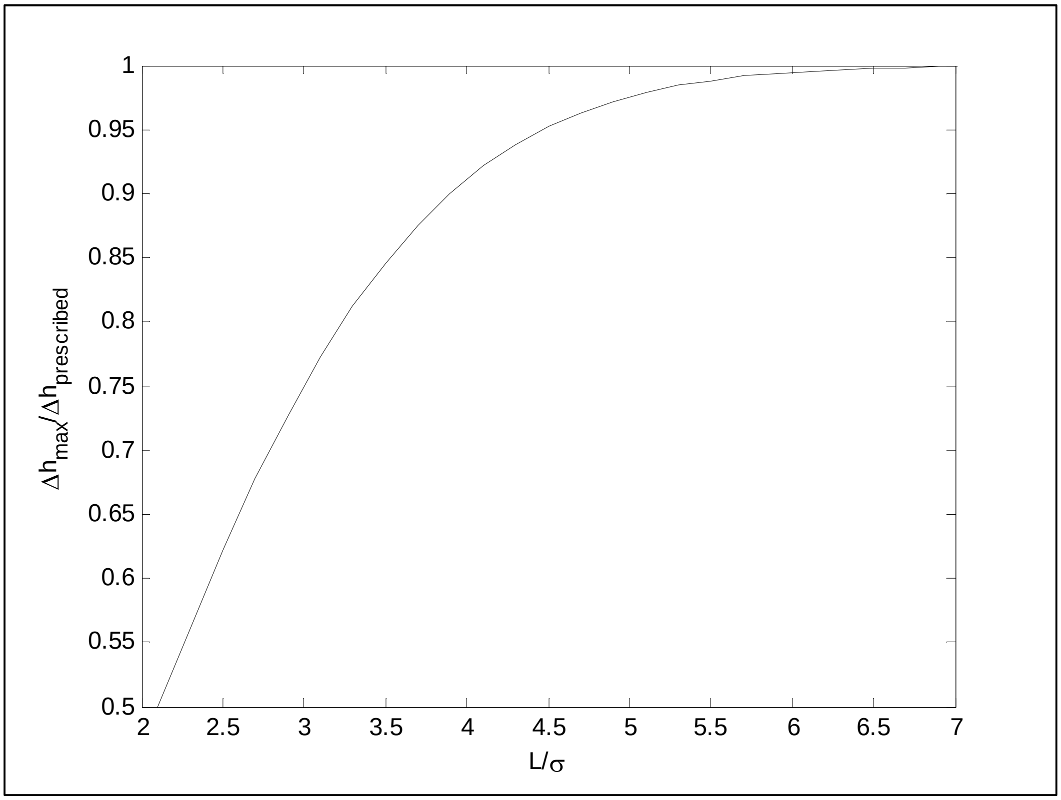
Researchers constructed a numerical experiment to determine the minimum size of a management zone such that it can be independently managed. They generated prescriptions that mimicked the case of an isolated area different from all of its surrounding neighbors.[5] They then convolved this prescription with Equation 4 (in Appendix A) and recorded the resultant best-possible application depth at the center of the management zone. The researchers used this general setup to vary the following features:

- Size of the management zones
- Relative difference in application depth between the isolated management zone and its surroundings
- Absolute magnitude of application depth
- Characteristic length scale

Figure 9 summarizes all results from this exploration.

[5] Only the center of the management zone must be independent of its neighbors. The remainder of an isolated management zone would consist of transitional regions.

Figure 9. Ratio of Best Achievable Application Depth Change to Prescribed Depth Change as a Function of the Management Zone Size and the Characteristic Length Scale of the Pivot



Notes: This nondimensional plot is used to determine the independence of an isolated management zone from its neighbors.

$\Delta h_{\max}$ = simulated maximum difference between the center of the isolated management zone and its surroundings

$\Delta h_{prescribed}$ = prescribed difference

L = length of the edge of a square management zone

σ = characteristic length scale

For the center of a management zone to be completely independent of its neighbors, the ratio $\Delta h_{\max} / \Delta h_{prescribed}$ would be 1. Figure 9 illustrates that the ratio $\Delta h_{\max} / \Delta h_{prescribed}$ approaches 1 as the ratio L / σ increases, and that when $\Delta h_{\max} / \Delta h_{prescribed}$ is 0.99, $L / \sigma = 5.6$. Taking $L / \sigma = 5.6$ and $\sigma = 4.15$ m, the smallest management zone achievable for the pivot under investigation is 23 m (13.6 feet). A ratio $\Delta h_{\max} / \Delta h_{prescribed}$ = 0.99 reflects an assumed acceptable level of performance and uniformity within each management zone, although Figure 9 also allows determination of minimum management zone sizes reflecting other depth ratios. Note that the minimum management scale is about twenty percent larger than the expected throw diameter of the Nelson R3000 brown plate sprinklers equipped on this pivot (throw diameter is about nineteen meters).

While the measured minimum management zone size is specific to this pivot with the current set of nozzles, the approach outlined above is extendable. Since the characteristic length scale does not depend on position, direction, or transition magnitude, it need be measured in only one place, in one direction at a single transition. That is, a standard catch-can test with a single line of catch cans arranged radially is sufficient as long as a single, detectable transition exists along this line. Researchers can then use Equation 3 (in Appendix A) to fit these data and determine the characteristic length scale. Given an acceptable cutoff ratio of $\Delta h_{\max} / \Delta h_{prescribed}$, they can then use the measured characteristic length scale in conjunction with Figure 9 to determine the minimum management zone size of a pivot under investigation.

4. Risks and Challenges

4.1. Risks

- Each VRI system is tailored to its location; therefore, although the numbers presented in this study are not directly transferable to every VRI pivot, the methodology presented is transferable. Determining values for all of the features and metrics described in this report requires performance of a complex catch-can test similar to the one conducted for this study, which included several elements not present in standard catch-can tests. The *Pivot Evaluation Best Practices* report describes a standard (simpler and less expensive) catch-can test of the type more typically used. Based upon the generalizable values of catch-can testing identified in this study, the research team recommends the inclusion of standard catch-can tests in a pivot evaluation process for VRI pivots, such as that described in the *Pivot Evaluation Best Practices* report.

4.2. Challenges

- Successful catch-can tests require attention to weather conditions to ensure reliable results. Researchers should refrain from conducting catch-can tests when the air temperature and consequently the evaporation level are both high. In addition, wind speed must be at or below the wind limitations specified in the American Society of Agricultural and Biological Engineers (ASABE) guidelines for catch-can testing (ASAE 2001).[6] Accommodating both temperature and wind speed in choosing times for catch-can testing will minimize the impact of weather conditions on the amount of water reaching each catch can.
- Researchers must be conscious of the farm’s operations when scheduling a catch-can test on a particular field. Coordinating with the grower’s schedule and actions planned for the field will help the researchers to maintain a positive relationship with the grower.
- Researchers need to consider the height of the crop when scheduling a catch-can test. The tops of the catch cans must remain above the plants so that water from the pivot is not diverted to the foliage.
- Uneven field terrain created by tillage and other conditions may at times impede the satisfactory execution of a catch-can test.
- Standard catch-can tests are not cheap, but cost-saving measures are available. Labor constitutes the largest proportion of the cost of a catch-can test; using GPS technology for catch can placement has streamlined the process, thus reducing the field portion of labor costs. The use of acceptable containers such as large yogurt tubs (about seventy-five cents each) for the catch cans reduces costs dramatically compared to the use of official off-the-shelf catch cans, which cost roughly $100 each.

[6] The American Society of Agricultural Engineers (ASAE) as of 2005 adopted its new name as the current American Society of Agricultural and Biological Engineers (ASABE).

5. Lessons Learned, Next Steps, Value of Findings

5.1. Lessons Learned

- Researchers calculated a minimum management zone size of twenty-three meters (seventy-five feet) in this study. Other VRI systems would exhibit different minimum management zone sizes, depending on their existing management zone sizes and sprinkler throws; however, the principles used to calculate the minimum management zone size in this study should apply to all of them.
- The original VRI system design for the field under investigation had management zones of approximately ten meters (thirty-three feet) on each edge. In the researchers' first attempt, they assumed a minimum management zone size of less than ten meters (thirty-three feet), and designed a prescription based on that assumption. That first test produced an unsuccessful outcome because the researchers' initial assumption did not accommodate the possibility of a larger minimum management zone size. The researchers redesigned the second attempt so that it would address the faulty assumption, and that prescription led to the set of findings presented in this report.
- The performance coefficient used to evaluate the effectiveness of VRI showed diminished performance of the VRI system near step changes in the prescribed water application.
- The new performance coefficient the researchers computed using the best-achievable prescription identified through the steps described in Section 3 demonstrated a significant improvement in agreement between the measured and prescribed (best-achievable) water depths, even in areas of transition.

5.2. Next Steps

The results of this study provided researchers sufficient information (keeping in mind the small sample size and variations among fields and irrigation systems) to develop the following directional recommendations for vendors and utilities.

<u>Recommendations to Vendors</u>

- The findings in this study imply that researchers, agronomists, and consultants can use the characteristic length to guide the physical design of the pivot VRI system (sprinkler and valve layout), as well as the irrigation prescription (optimized map of applied depths). The resultant optimization of the VRI technology should maximize improved precision in water distribution, allowing application to be matched to variability in field conditions, which can save water and energy and improve yield and profit.
- The study team offers the following additional considerations:
    - Refrain from making "pixelated prescriptions" and keep each area of management within the prescription greater than the minimum management zone size.
    - When in doubt, use the convolution approach to see a more realistic prediction of applied depth.

Recommendations to Utilities

- Using a traditional catch-can test with a 0-100% transition in the radial direction when conducting VRI pivot evaluations will help in determining pivot performance.
- The new “performance coefficient” is a direct analog to the “uniformity coefficient” used in Precision Flat Rate irrigation commissioning, and will provide a useful measure of VRI system performance. The *Pivot Evaluation Best Practices* report addresses topics relevant to both of these recommendations.

5.3. Value of Findings

- Application of the approach outlined in this report can improve VRI systems by eliminating unnecessary equipment, planning, and maintenance associated with overly-complex designs. Researchers can streamline this approach so they can measure a characteristic length scale with a traditional catch-can test under a VRI system. Once they know the characteristic length scale of the pivot, they can find the minimum management zone scaling with direct application of the type of plot exemplified in Figure 9.
- Researchers can more realistically assess a VRI system’s performance, and therefore estimate its potential efficiency gains, with a performance coefficient that reflects the actual depth of irrigation rather than ideal prescriptions.

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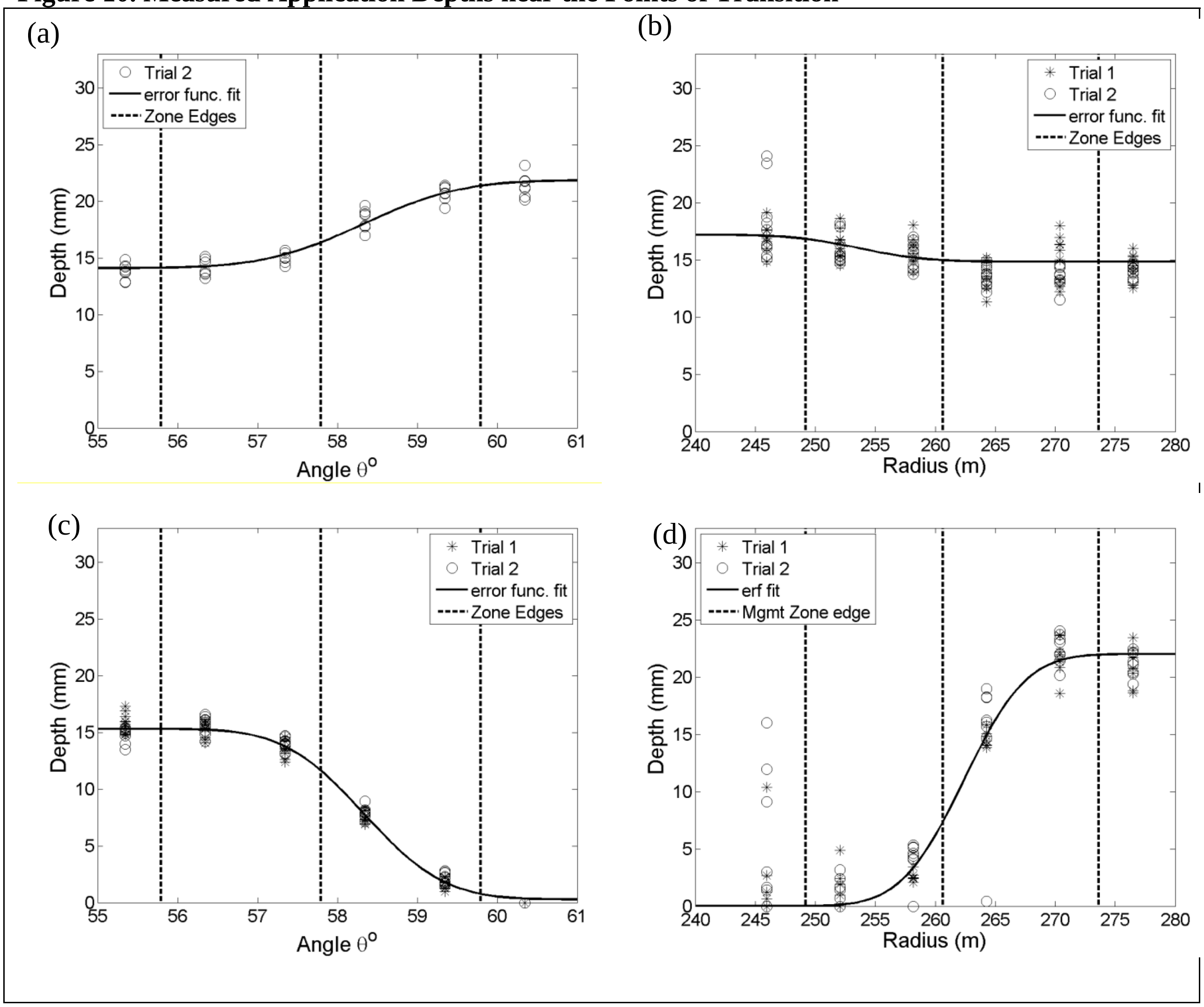
Appendix A – Determining the Characteristic Length of the Water Application

Measured water depths near the transitions in the prescribed water depth (both in the radial and angular directions) were isolated and analyzed. Data points within 10m of two-dimensional transitions, such as those at the very center of the measurement array, are not included in the analysis. Figure 10 presents these data. In each case, the data tend to track a smooth profile from one constant application depth to another. This behavior suggests that a redistribution process similar to dispersion may be the underlying mechanism at transitions. Just as the throw diameter of a nozzle defines an area receiving an acceptably uniform application depth, the solution to the diffusion equation describes the pattern of dispersal observed in the transitions.

Figure 10. Measured Application Depths near the Points of Transition



Notes: (a) transition from 50% to 80%, (b) transition from 60% to 50%, (c) transition from 60% to 0%, and (d) transition from 0% to 80%. The solid black lines are the empirical fits (using nonlinear least square optimization). Dashed lines are the boundaries between management zones.

A transition in a single dimension that is driven by a dispersive process would be described by the function

$$\frac{D(x) - D_{\min}}{D_{\max} - D_{\min}} = \frac{1}{2}\left(1 + erf\left(\frac{x-a}{\sqrt{2\sigma^2}}\right)\right), \tag{3}$$

where $D(x)$ is the application depth as a function of position (x), $D_{\max}$ is the maximum application depth across the transition, $D_{\min}$ is the minimum application depth across the transition, a is the position of the transition, and σ is the characteristic length of the transition.

This is a characteristic length in same sense that the standard deviation describes the half height and half width of a Gaussian distribution. In other words, an area spanned by a radius of three characteristic lengths contains 99% of the distribution. By defining the wetted area in terms of characteristic length, a spatial distribution (with a known confidence interval) can be determined from the catch can samples of measured depth. Note that advection (wind drift in this case) is neglected in Equation 3. If wind drift is significant, it can be included in Equation 3 with a variable transform following (Fischer 1979). Nonlinear least squares optimization (Matlab function “nlinfit’) is used to find the optimal value of σ for each one-dimensional transition. Table 1 presents the values of σ found for each transition, and Figure 10 above shows the resulting function fits for each case.

Table 1. Values of the Transition Length Scales as Determined by Fitting Equation 3 to the Data

Transition	Orientation	Length scale, σ
60% → 50%	Radial	4.4 m
50% → 80%	Angular	5.0 m
60% → 0%	Angular	3.1 m
0% → 80%	Radial	4.1 m

Note: Plots of the data and function fits are shown in Figure 10

The average of these length scales is 4.15 m (13.6 feet). From this analysis, there is no discernible dependence on the behavior of the transition with the magnitude of the transition. Indeed, the dispersion profile is non-dimensionalized explicitly to show the mathematical underpinnings of this independence (see Equation 3). The summary results suggest that there may be a dependence of the transition length scale on the radial position; however, when the transition length scale for each arc (all at differing radii) is computed independently, no pattern emerges to suggest that the length scale depends on radial position (results not shown). Thus from this point forward, this report uses a constant value of σ =4.15 m (13.6 feet) in calculations.

Following that the pivot is incapable of producing sharp transitions as prescribed due to the sprinkler pattern and wind dispersion, the next step is to translate the effects of the measured characteristic length scale into a best achievable application scenario. In other words, given the amount of smoothing observed in the one-dimensional transitions, what is the best achievable implementation of the two-dimensional prescription? Translation from one-dimension to two-dimensions is accomplished by up scaling from a point source solution for the dispersion profile. The underlying two-dimensional dispersion relationship for an isolated point source follows a two-dimensional Gaussian curve (written in Cartesian coordinates),

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{x^2 + y^2}{2\sigma^2} \right), \tag{4}$$

where σ is the measured characteristic length scale from the one-dimensional transitions.